Let $b^2\cos^2 C-b^2+c^2=1^2$. Then $c^2=1^2+b^2\sin^2 C$. Assume $1=x^2-y^2$, and $b\sin C=2xy$. Then $a=\sqrt{(b^2-4x^2y^2)}\pm(x^2-y^2)$.

Let $b^2-4x^2y^2=s^2$, $b+2xy=ks$, and $b-2xy=s/k$. Then $b=s(k^2+1)/2k$, $xy=s(k^2-1)/4k$.

Taking $s=4mk$ we obtain the following formulae:

$$b=2m(k^2+1),\quad c=x^2+m^2(k^2-1)^2/x^2,$$
$$a=4mk\pm[x^2-m^2(k^2-1)^2/x^2],\quad C=\cos^{-1}[2k/(k^2+1)].$$

As an example, assume $x=3$, $m=3$, $k=2$. We find $a=24$, $b=30$, $c=18$, $\cos C=\frac{4}{5}$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

m^2+n^2, m^2-n^2, $2mn$ are the sides of any right triangle. Let $m=pn$.

$\therefore n^2(p^2+1)$, $n^2(p^2-1)$, $2n^2p$ are the sides.

The equation for the general value of θ is

$$n^2(p^2-1)\cos\theta+2n^2p\sin\theta=n^2(p^2+1).$$

Let $\sin\beta=\dfrac{n^2(p^2-1)}{n^2(p^2+1)}$, $\cos\beta=\dfrac{2n^2p}{n^2(p^2+1)}$, then the equation becomes

$$\cos\beta\,\cos\theta+\sin\beta\,\sin\theta=1,\ \cos(\theta-\beta)=1.$$

$\therefore\theta-\beta=2\pi n_1$, $\theta=2\pi n_1+\beta$.

As θ must be less than $\frac{1}{2}\pi$, the general value of θ is $\theta=\beta$.

Also solved by J. B. Greggs, M. Sc., C. E., Senecaville, Ohio.

PROBLEMS FOR SOLUTION.

ALGEBRA.

192. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, O.

What is the difference between the squares of the two *infinite* continued fractions $\left(3+\dfrac{1}{6+\text{etc.}}\right)$ and $\left(2+\dfrac{1}{4+\text{etc.}}\right)$?

193. Proposed by SAUL EPSTEEN, Ph. D., Chicago, Ill.

Professor Goursat states (*Transactions of the American Mathematical Society*, January, 1904, p. 111) that if $a_1, a_2, \ldots, a_n$; $h_1, h_2, \ldots, h_n$ are two sequences, the h's being all positive, then $\sum\dfrac{a_i^2}{h_i^2}\geqq\dfrac{(\Sigma a_i)^2}{\Sigma h_i}$. Prove this.

194. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

In the determination of the canonical forms of Abelian transformations modulo p, one is led to the type $[b_1, b_2, b_3]$:

$$\xi_1'=\xi_1,\ \eta_1'=b_1\xi_1+\eta_1+b_2\xi_2+\eta_2+b_3\xi_3+\eta_3,\ \xi_2'=\xi_2-\xi_1,$$
$$\eta_2'=\eta_2+b_2\xi_2,\ \xi_3'=\xi_3-\xi_1,\ \eta_3'=\eta_3+b_3\xi_3.$$

Find its period and determine the conditions under which it is conjugate with $[c_1, c_2, c_3]$ under Abelian transformation.

GEOMETRY.

219. Proposed by H. F. MacNEISH, A.B., Assistant in Mathematics, University High School, Chicago, Ill.

Draw a line through a given point which shall divide a given quadrilateral into two equivalent parts; (1) when the point lies in a side of the quadrilateral, (2) when the point is without, (3) within the quadrilateral.

220. Proposed by G. B.M. ZERR, A. M., Ph. D., Parsons, West Va.

Two triangles are circumscribed to a given triangle ABC, having their sides perpendicular to the sides of the given triangle. Prove that the two triangles are equal, and find the area of these triangles.

DIOPHANTINE ANALYSIS.

120. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Find the prime numbers p for which $x^2-pxz-px-z+p^2-3=0$ has more than two sets of positive integral solutions x, z, each $<p$.

AVERAGE AND PROBABILITY.

152. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

A square hole is cut through the center of a sphere of radius r. Show that the average volume removed is $\frac{1}{12}r^3(23\sqrt{2}-28)$.

MISCELLANEOUS.

143. Proposed by R. L. MOORE, A. M., Fellow in Mathematics, The University of Chicago.

Does there exist a function, of single valued inverse, satisfying the following conditions: (1) if $u=f(v)$, $w=f(u)$ then $w=f(v)$; (2) given $u_1=f(u)$, $v_1=f(v)$, $w_1=f(w)$; if the triple w, v, w can be regarded as the lengths of the sides of a triangle, the triple u_1, v_1, w_1 cannot be similarly regarded.

144. Proposed by IRA M. DeLONG, Professor of Mathematics in the University of Colorado, Boulder, Col.

Determine the number of distinct spherical polygons of n sides formed by arcs of n given great circles on a sphere, each arc to be greater than 0 degrees, and less than 360 degrees, and no two sides of any polygon to lie an the same circle.